MATH4010 Functional Analysis (2020-21): Homework

Important Notice:

♣ The answer paper must be submitted before the deadline.
♠ The answer paper MUST BE sent to the CU Blackboard. Please refer to the course web for details.
✠ Each answer paper must include your name and student ID.

Homework 1. Deadline: 21 Sep 2020

1. Let X be the space of all bounded continuous real-valued functions defined on $\mathbb{R}$. For each $f \in X$, put $\|f\|_\infty := \sup\{|f(x)| : x \in \mathbb{R}\}$.

   (a) Show that X is a Banach space under the sup-norm $\|\cdot\|_\infty$.

   (b) Put $Y := \{f \in X : f$ is differentiable on $\mathbb{R}\ \}$. Is Y a closed subspace of X?

   (c) Let $Z := \{f \in X : $ the Riemann improper integral $\int_{-\infty}^{\infty} |f(x)| dx$ exists$\}$. Also, for each $f \in Z$, let $\|f\|_1 := \int_{-\infty}^{\infty} |f(x)| dx$. Are the norms $\|\cdot\|_1$ and $\|\cdot\|_\infty$ equivalent on Z?

$* * *$ **End** $* * *$